Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 1: Pure Mathematics

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Time: 2 hours	Paper Reference(s)
	9MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 14 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions. Write your answers in the spaces provided.

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{\theta \tan 3\theta}{\cos 2\theta - 1}$$

(Total for Question 1 is 3 marks)

2.

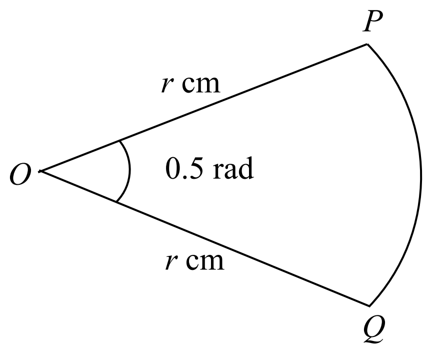

Figure 1

Figure 1 shows a sector POQ of a circle with centre O and radius r cm.

The angle POQ is 0.5 radians.

The area of the sector is 9 cm^2.

Show that the perimeter of the sector is k times the length of the arc, where k is an integer.

(Total for Question 2 is 4 marks)

3. The curve C has equation

$$y = 8\sqrt{x} + \frac{18}{\sqrt{x}} - 20 \qquad x > 0$$

a. Find

i) $\frac{dy}{dx}$

ii) $\frac{d^2y}{dx^2}$

(3)

b. Use calculus to find the coordinates of the stationary point of C.

(4)

c. Determine whether the stationary point is a maximum or minimum, giving a reason for your answer.

(2)

(Total for Question 3 is 9 marks)

4. The curve with equation $y = 2 + \ln(4 - x)$ meets the line $y = x$ at a single point, $x = \beta$.

a. Show that $2 < \beta < 3$

(2)

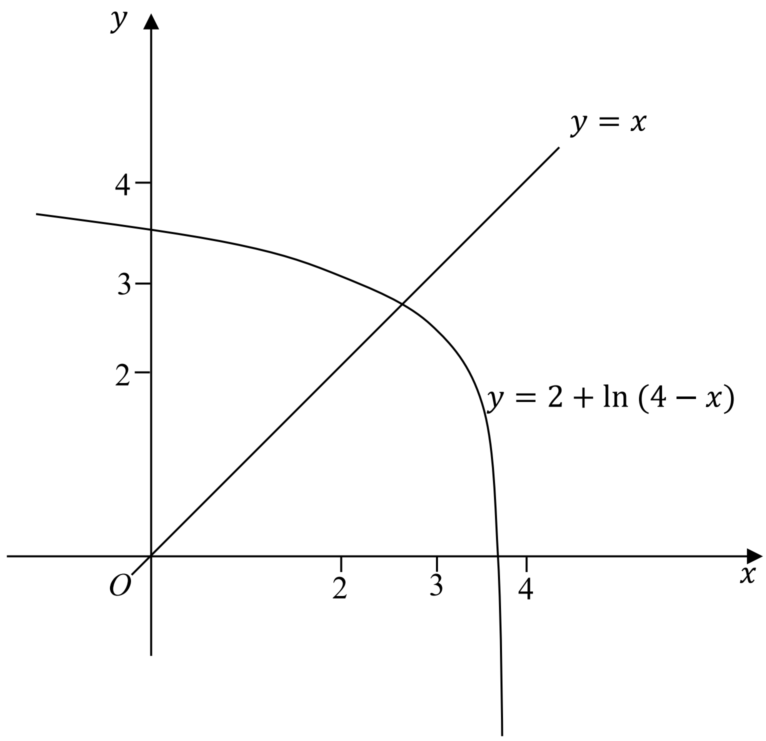

Figure 2

Figure 2 shows the graph of $y = 2 + \ln(4 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2 + \ln(4 - x_n), \qquad n \in N,$$

in an attempt to find an approximation for β.

Using the graph and starting with $x_1 = 3$,

b. determine whether the or not this iteration formula can be used to find an approximation for β, justifying your answer.

(2)

(Total for Question 4 is 4 marks)

5. Given that

$$y = \frac{5\cos\theta}{4\cos\theta + 4\sin\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

Show that

$$\frac{dy}{d\theta} = -\frac{5}{4(1+\sin 2\theta)}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

(Total for Question 5 is 5 marks)

6.

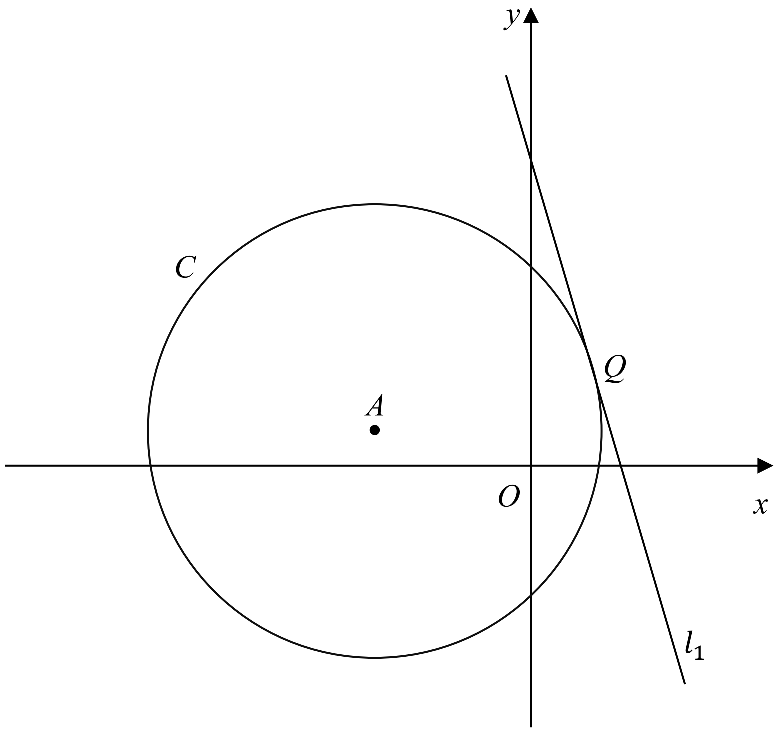

Figure 3

The circle C has centre A with coordinates $(-3,1)$.

The line l_1 with equation $y = -4x + 6$, is the tangent to C at the point Q, as shown in Figure 3.

a. Find the equation of the line AQ in the form $ax + by = c$.

(3)

b. Show that the equation of the circle C is $(x+3)^2+(y-1)^2=17$

(4)

The line l_2 with equation $y = -4x + k,\ k \neq 6$, is also a tangent to C.

c. Find the value of the constant k.

(3)

(Total for Question 6 is 10 marks)

7. Given that $k \in \mathbb{Z}^+$

a. show that $\int_{2k}^{3k} \frac{6}{(7k-2x)}\,\mathrm{d}x$ is independent of k,

(4)

b. show that $\int_{k}^{2k} \frac{2}{3(2x-k)^2}\,\mathrm{d}x$ is inversely proportional to k.

(3)

(Total for Question 7 is 7 marks)

8. The length of the daylight, $D(t)$ in a town in Sweden can be modelled using the equation

$$D(t) = 12 + 9\sin\left(\frac{360t}{365} - 63.435\right) \qquad 0 \leq t \leq 365$$

where t is the number of days into the year, and the argument of $\sin x$ is in degrees

a. Find the number of daylight hours after 90 days in that year.

(1)

b. Find the values of t when $D(t) = 17$, giving your answers to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable*)

(4)

(Total for Question 8 is 5 marks)

9.

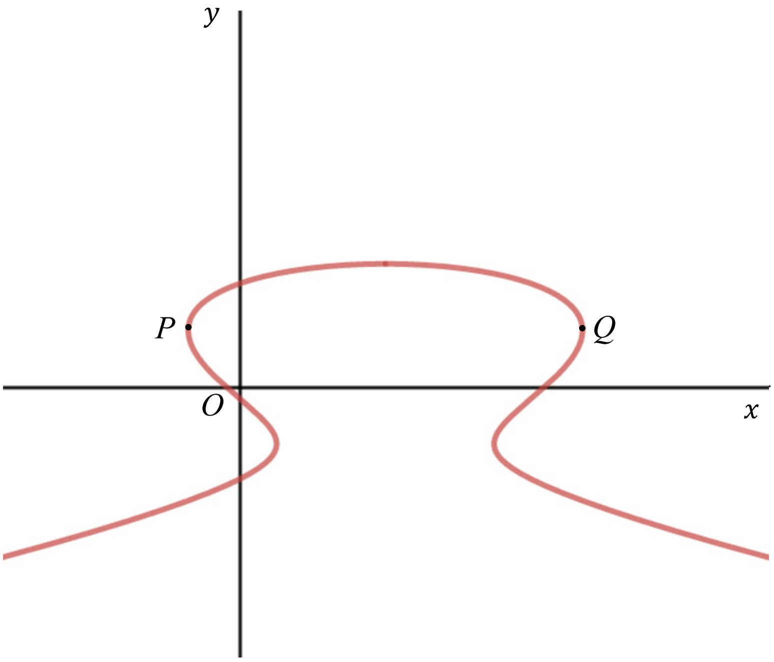

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 + y^3 - 10x - 12y - 5 = 0$

a. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10-2x}{3y^2-12}$

(4)

At each of the points P and Q the tangent to the curve is parallel to the y-axis.

b. Find the exact coordinates of Q.

(4)

(Total for Question 9 is 8 marks)

10. a. Find $\int \frac{1}{30} \cos \frac{\pi}{6} t \quad dt$.

(2)

The height above ground, X metres, of the passenger on a wooden roller coaster can be modelled by the differential equation

$$\frac{dX}{dt} = \frac{1}{30} X \cos(\frac{\pi}{6} t)$$

where t is the time, in seconds, from the start of the ride.

At time $t = 0$, the passenger is 6 m above the ground.

b. Show that $X = ke^{\frac{1}{5\pi}\sin(\frac{\pi}{6}t)}$ where the value of the constant k should be found.

(4)

c. Show that the maximum height of the passenger above the ground is 6.39 m.

(1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

d. Find the value of T.

(2)

(Total for Question 10 is 9 marks)

11. a. Find the binomial expansion of $(4 - x)^{-\frac{1}{2}}$, up to and including the term in x^2.

(4)

Given that the binomial expansion of $f(x) = \sqrt{\frac{1+2x}{4-x}}$, $|x| < \frac{1}{4}$, is

$$\frac{1}{2} + \frac{9}{16}x - Ax^2 + \cdots$$

b. Show that the value of the constant A is $\frac{45}{256}$

(3)

c. By substituting $x = \frac{1}{4}$ into the answer for (b) find an approximate for $\sqrt{10}$, giving your answer to 3 decimal places.

(3)

(Total for Question 11 is 10 marks)

12. The table shows the average weekly pay of a footballer at a certain club on 1 August 1990 and 1 August 2010.

Year	1990	2010
Average weekly pay	£2500	£50000

The average weekly pay of a footballer at this club can be modelled by the equation

$$P = Ak^t$$

where £P is the average weekly pay t years after 1 August 1990, and A and k are constants.

a. i. Write down the value of A.

(1)

ii. Show that the value of k is 1.16159, correct to five decimal places.

(2)

b. With reference to the model, interpret

i. the value of the constant A,

ii. the value of the constant k,

(2)

Using the model,

c. find the year in which, on 1 August, the average weekly pay of a footballer at this club will first exceed £100000.

(4)

(Total for Question 12 is 9 marks)

13.

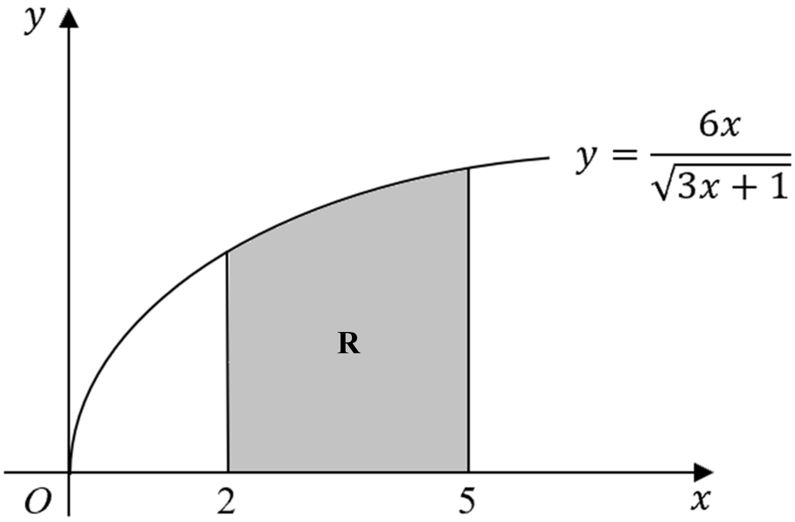

Figure 5 shows a sketch of part of the curve with equation $y = \frac{6x}{\sqrt{3x+1}}$, $\quad x \geq 0$

The finite region **R**, shown shaded in figure 5 is bounded by the curve, the x-axis and the lines $x = 2$ and $x = 5$.

Use the substitution $u = 3x + 1$ to find the exact area of **R**.

(Total for Question 13 is 7 marks)

14. A curve C has parametric equations

$$x = 1 - \cos t\,, \qquad y = 2\cos 2t, \qquad 0 \leq t < \pi$$

a. Show that the cartesian equation of the curve can be written as $y = k(1-x)^2 - 2$ where k is an integer.

(2)

b. i. Sketch the curve C.

ii. Explain briefly why C does not include all points of $y = k(1-x)^2 - 2\,,\ \ x \in \mathbb{R}$.

(3)

The line with equation $y = k - x$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

(Total for Question 14 is 10 marks)